The Λ-Single Particle Energy Levels in $^{16}_{\Lambda}O$ Hypernucleus

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Abstract

The purpose of this research is to investigate Λ single particle energy levels of $^{16}_{\Lambda}O$ hyper-nucleus by assuming that single Λ moves independently in an averaged potential well which has the Woods-Saxon form including spin-orbit interaction. The radial Schrödinger equation is solved numerically by using Numerov Method. The calculated results are compared with other theoretical results. The calculated results of Λ single particle energy levels of $^{16}_{\Lambda}O$ hyper-nucleus are in good agreement with other theoretical results.

Key words: Numerov Method, Energy levels.

Introduction

The nuclear shell model is one of the most important and useful models of nuclear structure. The shell model of the nucleus assumes that the energy structure (energy levels of the nucleons) of the nucleus is similar to that of an electron shell in an atom. According to this model, the protons and the neutrons are grouped in shell in the nucleus similar to extra-nuclear electrons in various shells outside the nucleus. The shells are regarded as "filled" when they contain a specific number of protons or neutrons or both. The number of nucleons in each shell is limited by Pauli exclusion principle. The shell model is sometimes referred to as the independent particle model because it assumes that each nucleon moves independently of all the other nucleons and is acted on by an averaged nuclear field produced by the action of all the other nucleons.

Usually, the averaged field is represented by a simple harmonic oscillator potential. With this potential the energy levels are E_N= (N+3/2) ħ with $(N = 2n + \ell + 3/2)$ where n is the principle quantum number and ℓ is orbital quantum number. When each shell is completely filled with nucleons, it is called closed shell. The number of nucleons which make closed shell are called magic numbers [1]. The magic numbers are n = 2, 8, 20, 28, 50, 82, 126…. The energy levels corresponding to each value of ℓ for the nucleon are represented by a spectroscopic notation similar to the electrons, ℓ = 0, 1, 2, 3, 4, 5,… for s, p, d, f, g respectively.

Spin –Orbit Potential

It was fairly clear by the 1940 that a central potential could not reproduce all the magic numbers. The crucial breakthrough came in 1940 when Maria Goeppert Mayer and

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Hans Jensen suggested – once again following the lead from atomic physics-that inside the nucleus, in addition to the central potential sensed by a nucleon has the form

$$V_{TOT} = V(r) - f(r)\vec{L}.\vec{S}, \tag{1}$$

where $\vec{L}$ and $\vec{S}$ are the orbital and the spin angular momentum operators for a nucleon, and f(r) is an arbitrary function of the radial coordinate [2,3,4]. In atomic physics, a spin-orbit interaction splits the two degenerate $j = l \pm \frac{1}{2}$ energy level and produces a fine structure. The total angular momentum operator is

$$\vec{J} = \vec{L} + \vec{S}, \tag{2}$$

Thus, in a state with definite $\ell, s,$ and j values, we have

$$\left\langle \vec{L}.\vec{S} \right\rangle = \frac{\hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right] \tag{3}$$

$$\left\langle \vec{L}.\vec{S} \right\rangle = \frac{\hbar^2}{2}\ell \qquad \text{for} \quad j = \ell + \frac{1}{2}$$

$$\left\langle \vec{L}.\vec{S} \right\rangle = -\frac{\hbar^2}{2}(\ell+1) \qquad \text{for} \quad j = \ell - \frac{1}{2}$$

where we have substituted $s = \frac{1}{2}$ for the spin of a nucleon.

Phenomenological Wood-Saxon Nucleon-Nucleus Potential

To investigate single-particle energy levels of ordinary nuclei, we emply the phenomenological Wood-Saxon nucleon-nucleus potential, which is as follow,

$$V(r) = -V_0 f(r) + V_{so}\left[\frac{\hbar}{m_\pi c}\right]^2 (\vec{L}.\vec{S})\frac{1}{r}\frac{df(r)}{dr} \tag{4}$$

where, f(r) = nuclear density $= \frac{1}{1 + e^{[(r-R)/a]}}$

For $j = \ell + \frac{1}{2}$,

$$V_{(r)} = -\frac{V_0}{1+\exp\left((r-R)/a\right)} - V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \left(\frac{1}{2}\ell\right)\left(\frac{1}{r}\right)\left(\frac{1}{a}\right)\frac{\exp\left((r-R)/a\right)}{\left(1+\exp\left((r-R)/a\right)\right)^2} - \tag{5}$$

For $j = \ell - \frac{1}{2}$,

$$V_{(r)} = -\frac{V_0}{1+\exp\left((r-R)/a\right)} + V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{2}(\ell+1)\left(\frac{1}{r}\right)\left(\frac{1}{a}\right)\frac{\exp\left((r-R)/a\right)}{\left(1+\exp\left((r-R)/a\right)\right)^2} \tag{6}$$

Numerov Method

The Shrödinger Radial Equation (SRE) can be written as

$$\frac{d^2U(r)}{dr^2} + k(r)u(r) = s(r) \qquad (7)$$

If l=0, $k(r) = \frac{2\mu}{\hbar^2}[E - V(r)]$

where k(r) = kernel of the equation s (r)=0

First we split the r range into N points according to $r_n = r_{n-1} + h$ (where h is the step); then we write the wavefunction

$u_n = u(r_n) = u(r_{n-1} + h)$, and

$k_n = k(r_n) = k(r_{n-1} + h)$.

Then approximating the second derivative by the three-point difference formula, and using it within the second-order differential equation we get the following recursive formulas, with a local error $0(h^6)$:

(a) Forward Recursive Relation,

$$u_n = \frac{2\left[1 - \frac{5h^2}{12}k_{n-1}\right]u_{n-1} - \left[1 + \frac{h^2}{12}k_{n-2}\right]u_{n-2}}{\left(1 + \frac{h^2}{12}k_n\right)} \qquad (8)$$

(b) Backward Recursive Relation,

$$u_{n-1} = \frac{2\left[1 - \frac{5h^2}{12}k_n\right]u_n - \left[1 + \frac{h^2}{12}k_{n+1}\right]u_{n+1}}{\left(1 + \frac{h^2}{12}k_{n-1}\right)} \qquad (9)$$

We have calculated the wave function using the backward-forward technique. In our calculation, two initial values for each direction are needed for the recursive formulas.

It is also necessary to know the first derivative at the appropriate order and as follows.

$$u'(r_n) = \frac{1}{2h}\left[\left(1 + \frac{h^2}{6}k_{n+1}\right)u_{n+1} - u_{n-1}\left(1 + \frac{h^2}{6}k_{n-1}\right)\right] \qquad (10)$$

Bound state energy

Since both $u_{out}(r)$ and $u_{in}(r)$ satisfy an homogeneous equation, they are set to be equal at the r_c point. An energy eigenvalue is then signaled by the equality of derivatives at this r_c point [5,6,7]. At the matching point the eigenfunctions $u_{out}(r)$ and $u_{in}(r)$ and first derivatives $u'_{out}(r)$ and $u'_{in}(r)$ must all satisfy the continuity conditions:

At the matching point

$$\left(U_{out}\right)_{rc} = \left(U_{in}\right)_{rc} \qquad (11)$$

$$\left(U'_{out}\right)_{rc} = \left(U'_{in}\right)_{rc} \tag{12}$$

$$\left[\frac{U'_{out}}{U_{out}}\right]_{rc} = \left[\frac{U'_{in}}{U_{out}}\right]_{rc} \tag{13}$$

Splitting this E range into N points according to

$$E_n = E_{n-1} + \Delta E \tag{14}$$

where ΔE = energy step

Matching eigen functions at the r_c point $U_{out}(r)$ and $U_{in}(r)$ eigen functions

$$U_{out}(r) = A\Phi(r) \qquad U_{in}(r) = BI(r)$$

Their respective derivative are

$$U'_{out}(r) = A\Phi'(r) \qquad U'_{in}(r) = BI'(r)$$

$$\left(A\Phi\right)_{rc} = \left(BI\right)_{rc} \qquad (A\Phi')_{rc} = \left(BI'\right)_{rc}$$

$$A = \left[\frac{I - I'}{\Phi - \Phi'}\right]_{rc} B = f_c B \tag{15}$$

$$U_{out}(r) = Bf_c\Phi(r) \qquad U_{in}(r) = BI(r)$$

where f_c = scaling factor

B = global factor

Normalization

Using normalization condition, we find $U_{out}(r)$ and $U_{in}(r)$ eigen functions

$$\int_0^{r_{max}} dr\left|u_l(r)\right|^2 = 1 \tag{16}$$

By separating the $U_{out}(r)$ and $U_{in}(r)$

$$\int_0^{r_{max}} dr\left|u_l(r)\right|^2 = \int_0^{rc} dr\left|u_{out}(r)\right|^2 + \int_{rc}^{r_{max}} dr\left|u_{in}(r)\right|^2 = 1 \tag{17}$$

$$\int_0^{r_{max}} dr\left|u_l(r)\right| = (Bf_c)^2 \int_0^{rc} dr\left|\Phi(r)\right|^2 + B^2 \int_{rc}^{r_{max}} dr\left|I(r)\right|^2 \tag{18}$$

$$B^2\left[f_c{}^2 \int_0^E dr\left|\Phi(r)\right|^2 + \int_{rc}^{r_{max}} dr\left|I(r)\right|^2\right] = 1 \tag{19}$$

Deriving the normalized eigen functions

$$U_{out}(r) = \frac{1}{\sqrt{N}} f_c \Phi(r)$$

$$U_{in}(r) = \frac{1}{\sqrt{N}} I(r)$$

where $B = \frac{1}{\sqrt{N}}$, normalization constant.

Results and Discussion

The Λ Single particle Energy Levels in $^{16}_{\Lambda}O$ Hyper-Nucleus

We have investigated the Λ single particle energy levels in $^{16}_{\Lambda}O$ nuclei. Numerov method can be evaluated both energy eigen values and wave functions simultaneously. The Harmonic Oscillator wavefunction of $^{16}_{8}O$ for various states are shown in Figure 1 (a), (b) and (c).

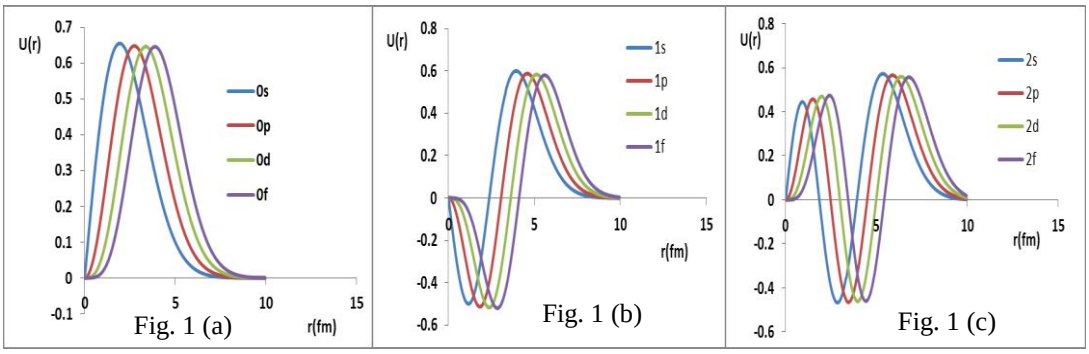

Fig. 1 (a) Fig. 1 (b) Fig. 1 (c)

Figure 1 (a), (b), (c) The Harmonic Oscillator wavefunction of $^{16}_{8}O$ for various states

We have also calculated the neutron single particle energy levels of $^{16}_{8}O$ for the Woods-Saxon potential including spin orbit interaction, which cannot be solved analytically. The calculated eigenvalues for the Woods-Saxon potential are listed in table 1.

Table 1. The neutron single energy levels in $^{16}_{8}O$ (MeV) for Woods-Saxon potential

State	without L.S	with L.S	
		J=l+1/2	J=l-1/2
0S	-32.49	-18.83	-32.49
0P	-17.74	-18.83	-15.58

And then, we have calculated Λ single particle energy levels in $^{16}_{\Lambda}O$ hypernucleus. $^{16}_{\Lambda}O$ hypernucleus contains single Λ particle and 15 nucleons. In this research, we

considered single Λ particle moves freely in an average field which is obtained from interaction between 15 nucleons and Λ according to shell model. Therefore we have calculated Λ- single particle energy level in ${}^{16}_{\Lambda}O$ hypernucleus for Woods-Saxon potential with spin-orbit coupling. The calculated eigenvalues of Λ single for the Woods-Saxon potential are listed in Table 2. The calculated results are compared with other theoretical results [8] in table 3. Our calculated results of Λ single particle energy levels are in good agreement with other theoretical results.

Table 2. The Λ single particle energy levels in ${}^{16}_{\Lambda}O$ (MeV) for Woods-Saxon potential

State	without L.S	with L.S	
		J=l+1/2	J=l-1/2
0S	-13.05	-13.05	-13.05
0P	-2.34	-2.76	-1.54

Table 3. Comparison of our calculate results of ${}^{16}_{\Lambda}O$ with other theoretical results

State	Our Calculated Results (MeV)	Other Theoretical Results (MeV) [8]
0s	- 13.05	- 13.0
$0p_{3/2}$	- 2.76	- 2.7
$0p_{1/2}$	-1.54	-1.5

Conclusion

The radial Schrödinger equation is solved numerically by using Numerov Method. We have calculated Λ single energy levels of ${}^{16}_{\Lambda}O$ hyper-nucleus. The calculated results are compared with other theoretical results. The calculated results of Λ single particle energy levels are in good agreement with other theoretical results.

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